Equilibrium states of C*-algebras associated to right LCM monoids

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Equilibrium states for C^*-algebras

KMS (equilibrium) states for a C^*-dynamical system $(\mathcal{A}, \sigma)$:

$$\sigma : \mathbb{R} \to \mathrm{Aut}(\mathcal{A}), \text{ time evolution,}$$

originate in mathematical physics/statistical mechanics. Some landmarks:

(i) Finite quantum systems and inner flows, 1960's.

(ii) Fermion algebra and approximately inner flows, 1970's.

(iii) $(\mathcal{A}, \sigma)$, σ not approximately inner flow, late 1970's.

Example from number theory with rich phase transition:

(iv) The *Bost-Connes* algebra, 1990's.

C^*-algebras from monoids with phase transition:

(v) The C^*-algebra of the affine semigroup of the rational numbers, 2010's.

(vi) A wealth of examples with $\mathcal{A}$ constructed from monoids in the past decade.

The KMS condition for finite systems

Finite quantum system: $\mathcal{A} = M_n(\mathbb{C})$ with (necessarily)

$$\sigma_t(A) = e^{itH} A e^{-itH},$$

where $t \in \mathbb{R}$, $A \in M_n(\mathbb{C})$ and H is a self-adjoint matrix.

For $\beta > 0$, the *Gibbs state* is $\varphi_G(A) = \frac{\mathrm{Tr}(Ae^{-\beta H})}{\mathrm{Tr}(e^{-\beta H})}$. It minimizes the free energy and satisfies

$$\varphi_G(A_1 A_2) = \varphi_G(A_2 \sigma_{i\beta}(A_1)), \tag{1}$$

for $A_1, A_2 \in M_n(\mathbb{C})$ *analytic*, i.e. $t \mapsto \sigma_t(A_j)$ extends to an entire function on $\mathbb{C}$, $j = 1, 2$.

Partition function of $(M_n(\mathbb{C}), \sigma)$ is $\beta \mapsto \mathrm{Tr}(e^{-\beta H})$.

The *KMS condition (1)*, cf. *Haag-Hugenholtz-Winninck* (1967) defines *equilibrium for a state* of a system $(\mathcal{A}, \sigma)$ at *inverse temperature* β. KMS for *Kubo-Martin-Schwinger*.

KMS states cf. Haag-Hugenholtz-Winninck

By analogy with finite systems and the Gibbs states, extend the notions of KMS_β state, partition function, inverse temperature.

$\mathcal{A}$ C^*-algebra, $\sigma : \mathbb{R} \to \mathrm{Aut}(\mathcal{A})$ time evolution, φ a state on A. Say that:

1. φ is KMS_β (at inverse temperature $\beta \in [0, \infty)$) if

$$\varphi(a_1 a_2) = \varphi(a_2 \sigma_{i\beta}(a_1))$$

   for all $a_1, a_2 \in \mathcal{A}$ with a_j in a dense subalgebra of *analytic* elements.

2. A state φ is a *ground state* if for all a_1, a_2 with a_2 analytic, the function $z \to \varphi(a_1 \sigma_z(a_2))$ is bounded in the upper-half plane.
3. KMS_∞ if $\varphi = w^* \lim \varphi_n$ as $\beta_n \to \infty$ and φ_n is KMS_{β_n}.

References: *Bratteli-Robinson*, *Pedersen*, *Connes-Marcolli*.

Systems without approx. inner flows

Olesen-Pedersen (1978), *Evans* (1980). KMS states for the extended *Cuntz* algebra $\mathcal{A} = C^*(\mathbb{F}_n^+)$, with $\mathbb{F}_n^+$ the free monoid on n generators. $\mathcal{A}$ is universal for n isometries $s_1, s_2, \ldots, s_n$ with $\sum_{j=1}^n s_j s_j^* < 1$. Time evolution determined by

$$\sigma_t(s_j) = e^{it} s_j, t \in \mathbb{R}, j = 1, \ldots, n.$$

Form $s_\mu = s_{j_1} \ldots s_{j_m}$ representing words $\mu = j_1 j_2 \cdots j_m$ in $\mathbb{F}_n^+$ of length $|\mu| = m \geq 1$. There is a conditional expectation E onto

$$D = \overline{\operatorname{span}}\{s_\mu s_\mu^* \mid \mu \in \mathbb{F}_n^+\},$$

a commutative C^*-subalgebra with spectrum $\widehat{D}$ the compactification of the space of finite paths in $\mathbb{F}_n^+$.
For $\beta \geq \log n$ there is a unique KMS_β *state s.t.*

$$\varphi_\beta(s_\mu s_\nu^*) = e^{-|\mu|\beta} \delta_{\mu,\nu},$$

lifted via E from the prob. measure $\delta_\mu \mapsto (1 - ne^{-\beta}) e^{-\beta|\mu|}$ above $\log n$. The "inverse temperature space": $[\log n, \infty)$.

C*-algebras from (left) cancellative monoids

Interrelated trajectories of development around KMS states:

(I) New classes of examples. Supply of examples grows, new opportunities.

(II) Identify common features from looking at disparate classes of monoids P and develop a common method for $C^*(P)$. Impetus comes from new classes of examples that do not fit existing paradigm.

Tractable $C^*(P)$: nuclear, interesting ideals and quotients.
As point of departure: $C^*(P)$ should have generators v_p with

- $v_p^* v_p = 1 = v_e$, i.e. isometry where $e \in P$ identity;
- $v_p v_p^* \leq 1$, i.e. not necessarily a unitary;
- $v_p v_q = v_{pq}, p, q \in P$, i.e. representation of P.

Semigroup C^*-algebras: *Nica* (1994), for P subsemigroup in a group forming a quasi-lattice order (e.g. $\mathbb{F}_n^+$ inside $\mathbb{F}_n$).

Monoids and equilibrium states: motivation

$\mathcal{A}$ a C^*-algebra, $\sigma : \mathbb{R} \curvearrowright \mathcal{A}$ a one-parameter group. Suppose

$$V^*V = 1_{\mathcal{A}}, VV^* < 1_{\mathcal{A}},$$

and $V \mapsto N(V) \in (0, \infty)$ s.t. $\sigma_t(V) = N(V)^{it} V$.

If $\varphi : \mathcal{A} \to \mathbb{C}$ is a KMS state at $\beta > 0$, then

$$\varphi(VV^*) = N(V)^{-\beta}\varphi(V^*V) = N(V)^{-\beta}.$$

For P a left-cancellative monoid, $\mathcal{A} = C^*(P)$ is generated, as a minimum, by isometries v_p with $v_p v_q = v_{pq}, p, q, \in P$. If $N : P \to (0, \infty)$ is a homomorphism (a *scale*), define $\sigma^N : \mathbb{R} \curvearrowright \mathcal{A}$

$$\sigma_t^N(v_p) = N_p^{it} v_p, p \in P.$$

An equilibrium state φ_β must have *prescribed values* $\varphi_\beta(v_p v_p^*)$, $p \in P$. Are there φ_β's? For what β's?

Affine monoids

Laca-Raeburn (2010). Consider $\mathbb{N} \rtimes \mathbb{N}^\times$ inside $\mathbb{Q} \rtimes \mathbb{Q}_+^\times$, with $(\mathbb{Q}, +)$ and $(\mathbb{Q}_+^\times, \cdot)$. Let $\mathcal{P}$ denote the set of primes. Then

$$C^*(\mathbb{N} \rtimes \mathbb{N}^\times) = C^*\langle s, v_\mathfrak{p}, \mathfrak{p} \in \mathcal{P} \mid \text{relations}\rangle$$

with time evolution σ^N where $N(s) = 1, N(v_\mathfrak{p}) = \mathfrak{p}$. The inverse temperature space consists of $[1, \infty]$ and ground states:

1. If $\beta \in [1, 2]$, there is a unique KMS$_\beta$ state.
2. If $\beta \in (2, \infty]$, the KMS$_\beta$ states are parametrised by probability measures on $\mathbb{T}$.
3. Ground states form a convex set isomorphic to the state space of $\mathcal{T}$, the Toeplitz C^*-algebra gen. by the unilateral shift on $\ell^2(\mathbb{N})$.
4. Partition function: $\zeta(\beta) := \sum_{n \in \mathbb{N}} n^{-\beta - 1}$, with pole at $\beta = 2$.

Crucial point: an expectation onto a comm. subalgebra with spectrum a "compactification" of a path space associated to the quasi-lattice order from $\mathbb{N} \rtimes \mathbb{N}^\times$ inside $\mathbb{Q} \rtimes \mathbb{Q}_+^\times$.

KMS states for monoid C^*-algebras

Generalisations to many q.l.o. (G, P): inverse temperature space (except ground states) of form $[1, \infty]$:

1. *Clark-an Huef-Raeburn*, Baumslag-Solitar monoids (certain one-relator monoids) that form quasi-lattice orders inside their groups.
2. *Afsar-Brownlowe-L-Stammeier*, *Brownlowe-L-Ramagge-Stammeier* for P right LCM, including Baumslag-Solitar monoids that form *weak* quasi lattice orders.
3. *Neshveyev-Stammeier*, uniqueness of φ_{β_c} at a critical value β_c using groupoid realisation of the C^*-algebras.

Key to (1)-(2): sufficiently special features of the *scale* $N : P \to (0, \infty)$, in particular $N(P) \subset \mathbb{N}^\times$ with some irreducibility properties.

But, scales on (most) Artin monoids fail these properties.

Right LCM monoids and equilibrium states

C*-algebras of monoids and their KMS states: We see algebraic structure from irreversibility of monoids in fruitful interaction with analytic features enabling existence of these special positive functionals φ with

$$\varphi(a_1 a_2) = \varphi(a_2 \sigma_{i\beta}(a_1))$$

for some real β and all a_1, a_2 analytic.

C*-algebras of right LCM monoids and their KMS states: The algebraic structure from irreversibility of monoids reflects, and is facilitated by, divisibility type properties in P.

C^*-algebras of monoids: reduced and full

Let P be a left-cancellative monoid. The *reduced* C^*-algebra is

$$C^*_\lambda(P) = C^*\langle T_p \mid T_p\varepsilon_q = \varepsilon_{pq}, p, q \in P\rangle \subset B(\ell^2(P)),$$

where $\{\varepsilon_p\}_p$ is the canonical o.n.b. in $\ell^2(P)$.

The *full/universal* C^*-algebra $C^*(P)$ should be generated by, as a minimum, elements v_p s.t., mirroring T_p's,

$$v_p^* v_p = 1, v_p v_p^* \leq 1, v_p v_q = v_{pq}, p, q \in P.$$

One tractable theory requires conditional least upper bounds under the partial order $p \leq r$ iff $r \in pP$.
Nica (1994): for $P \subset G$ a subsemigroup of a discrete group, $P \cap P^{-1} = \{e\}$, partial order

$$x \leq z \iff x^{-1}z \in P, \text{ for } x, z \in G,$$

require that a least common upper bound $x \vee y$ for $x, y \in G$, if it exists in P, is unique (quasi lattice orders). Extension to left cancellative P by *Li* (2012), *Norling* (2014), *Sehnem* (2019).

Some relevant classes of monoids

Quasi-lattice ordered pairs (G, P):

- (G, P), G totally ordered abelian with positive cone P (*Douglas, Murphy*);
- $(\mathbb{F}_n, \mathbb{F}_n^+)$ (*Nica*);
- right-angled Artin group-monoid pairs (G, P), (*Crisp-Laca*);
- affine monoids, e.g. $\mathbb{N} \rtimes \mathbb{N}^\times$, (*Laca-Raeburn*);
- Baumslag-Solitar monoids with matching signs, (*Spielberg, Clark-an Huef-Raeburn*);

Weak q.l.o. pairs, asking for $p \vee q$ to be unique in P, if it exists, only when $p, q \in P$:

- All Artin monoids (*Brieskorn-Saito*);
- Baumslag-Solitar monoids with opposite signs, (*Spielberg*).

Equilibrium states of $C^*_\lambda(P)$ for P q.l.o.

Theorem (Bruce-Laca-Ramagge-Sims (2018))

Let (G, P) be q.l.o and assume $N : P \to [1, \infty)$ satisfies $N(p) = 1$ only for $p = e$. Suppose that the Dirichlet series $\sum_{p\in P} N(p)^{-\beta}$ has abscissa of convergence $0 < \beta_c < \infty$. Define the generalised Gibbs state φ_β at $\beta > \beta_c$ on $B(\ell^2(P))$

$$\varphi_\beta(A) = \frac{\mathrm{Tr}(Ae^{-\beta H})}{\mathrm{Tr}(e^{-\beta H})},$$

where $H\varepsilon_p = \log N(p)\varepsilon_p$ on $\ell^2(P)$ (s.a. unbounded). Then, for $\beta > \beta_c$, φ_β is the unique KMS$_\beta$ state. At β_c there is a unique equilibrium state (obtained as a w^*-limit point of (φ_{β_n}), $\beta_n \to \beta_c$).

Unclear whether KMS$_\beta$ states exist for $\beta \in [0, \beta_c)$.

Right LCM (right least common multiples) monoids

P left cancellative monoid (or category).

1. $r \in P$ is a *right multiple* of $p \in P$ if $r = pp'$ for $p' \in P$.
2. $r \in P$ is a *common right multiple* of p, q in P if

$$r = pp' = qq' \text{ for } p', q' \in P.$$

3. P is right LCM (has conditional right LCM's) if every p, q with a common right multiple admit a least common right multiple, written $pP \cap qP \neq \emptyset$.

If P is right LCM, then $C^*(P)$ is *generated by isometries* v_p subject to $v_p v_q = v_{pq}$ and

$$v_q^* v_p = \begin{cases} v_{q'} v_{p'}^* & \text{if } pP \cap qP \neq \emptyset, \\ 0 & \text{if } pP \cap qP = \emptyset \end{cases}$$

for $pp' = qq' = r$, $p, q \in P$, (Clifford condition used by *Lawson*).

The class of right-angled Artin monoids

Suppose that Γ is a simple (undirected) graph, finite or infinite. The associated *right-angled Artin group* G_Γ is given by generators s_i indexed by the vertices of Γ and relations

$$s_i s_j = s_j s_i, \quad \text{if } i \text{ and } j \text{ share an edge.}$$

Let S_Γ the set of the generators s_i (the standard generating set). Denote by $P_\Gamma \subset G_\Gamma$ the monoid generated by S_Γ. Then (G_Γ, P_Γ) is q.l.o. and these objects interpolate between $(\mathbb{Z}^k, \mathbb{N}^k)$ and $(\mathbb{F}_n, \mathbb{F}_n^+)$ (*Crisp-Laca*).

Suppose that $N : P_\Gamma \to (0, \infty)$ is a monoid homomorphism. Define C^*-dynamical systems

$$(C^*(P_\Gamma), \sigma^N) \text{ and } (C^*_\lambda(P_\Gamma), \sigma^N),$$

where $\sigma^N(v_p) = N(p)^{it} v_p$ and $\sigma^N(T_p) = N(p)^{it} T_p$, $t \in \mathbb{R}, p \in P_\Gamma$. *Question: What are the KMS states?*

Equilibrium states for right-angled Artin monoids

Two approaches: by *Bruce-Laca-Ramagge-Sims and Afsar-L-Neshveyev*.

Let P_Γ be a non-abelian right-angled Artin monoid with finite generating set S_Γ. Let ℓ be normalised length function on P_Γ and set

$$N(p) = e^{\ell(p)}.$$

If the abscissa of convergence β_0 of $\sum_{p\in P} N(p)^{-\beta}$ is finite, then $(C^*(P_\Gamma), \sigma^N)$ has "inverse temperature space" $[\beta_0, \infty]$ (same for the reduced).

Question: What is the situation for other classes of Artin monoids, such as those of finite-type?

Need a new kind of insight than for previous classes of examples to see how KMS states could arise.

Equilibrium states and a positivity condition

Afsar-L-Neshveyev, (2019). Let (G, P) be a weak quasi-lattice order, $N\colon P \to (0, \infty)$ a homomorphism, and $(C^*(P), \sigma^N)$ the associated C^*-dynamical system. Let $\Phi : C^*(P) \to \mathcal{F}$ the conditional expectation onto the fixed point-algebra for the canonical coaction of G.

For each $\beta \in \mathbb{R}$, there is a Φ-invariant (factoring through Φ) KMS$_\beta$-state iff

$$(*) \sum_{K \subset J} (-1)^{|K|} N(q_K)^{-\beta} \geq 0 \text{ for all finite } J \subset P \setminus \{e\},$$

where $q_K = \bigvee_{q \in K} q$ whenever it exists in P, otherwise the term in the sum is set to 0.

Roughly, this constructs a KMS$_\beta$ state from prescribed values $N(q_K)^{-\beta}$. However, the condition $(*)$ involves an infinite collection of inequalities, so is unyielding.

The positivity for right-angled Artin monoids

Afsar-L-Neshveyev, (2019). Suppose P_Γ is a right-angled Artin monoid on a graph Γ with standard generating set S_Γ. Let $C(S_\Gamma)$ be the collection of finite cliques on the standard generators, i.e. the collection of sets of generators corresponding to complete finite subgraphs of Γ, and set $s_K = \prod_{s\in K} s$ for $K \in C(S_\Gamma)$.
There is a Φ-invariant equilibrium state at $\beta \in \mathbb{R}$ iff

$$(**) \quad \sum_{\substack{K\in C(S_\Gamma):\\ K\subset J}} (-1)^{|K|} N(s_K)^{-\beta} \geq 0 \text{ for all finite } J \subset S_\Gamma.$$

This positivity-reduction, i.e. going from sufficiency of $(*)$ to that of $(**)$, is used in the right-angled case to settle that the inverse temperature space is a half-line. Does positivity reduction hold for other P?

Artin monoids revisited

The Artin group associated to a Coxeter matrix $M=(m_{st})_{s,t\in S}$ is the group A_M with generating set S and presentation

$$\{S \mid \langle st\rangle^{m_{st}}=\langle ts\rangle^{m_{ts}} \text{ for all } s,t\in S\},$$

where $m_{st}=m_{ts}\in\{2,\dots,\infty\}$ for $s\neq t$. Let A_M^+ be the monoid with the same presentation. It is of *finite type* if the Coxeter group determined by the presentation for A_M together with $s^2=e$ for all s is finite. It is *right-angled* if m_{st} is 2 or ∞.

Fact: All A_M^+ are right- and left-Noetherian. A left-cancellative monoid P is *right-Noetherian* provided that there exists no infinite ascending sequence in P with respect to proper left-divisibility. Thus for $q\in P$, every sequence

$$q=p_1t_1=p_1p_2t_2=p_1p_2p_3t_3=\dots$$

with p_n, t_n non-invertible must terminate. Similar for left-Noetherian.

Characterising KMS states through positivity

Theorem (Gazdag-Laca-L (2022))

For P cancellative Noetherian right LCM and $N : P \to (0, \infty)$, existence of an expectation-invariant equilibrium state at β for $(C^*(P), \sigma^N)$ is determined by

$$(*) \sum_{K \subset J} (-1)^{|K|} N(\vee K)^{-\beta} \geq 0 \text{ for all finite } J \subset P \setminus P^*$$

with $N(\vee K)^{-\beta} = 0$ if $\vee K = \infty$ (not a clique).

The result makes no reference to an ambient group, and applies to all Artin monoids, not just the quasi-lattice ordered ones (the right-angled).

Reduction of the positivity condition

For the system $(C^*(P), \sigma^N)$, an expectation invariant KMS$_\beta$ state exists subject to

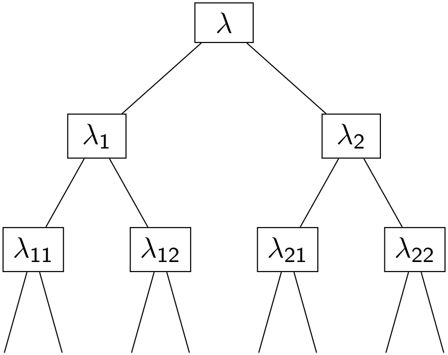
$$(*) \sum_{K \subset J} (-1)^{|K|} N(\vee K)^{-\beta} \geq 0 \text{ for all finite } J \subset P \setminus P^*$$

with $N(\vee K)^{-\beta} = 0$ if $\vee K = \infty$ (not a clique).

Goal: reduce verifying $(*)$ to verifying $(**)$ involving subsets J of the set P_a of atoms: elements $a \in P$ without decompositions $a = yz$ for $y, z \notin P^*$.

Idea: from $J = \{p_1, \dots, p_n\}$, "remove" atoms a from its elements, in a two-step procedure leading to two new families J_1 and J_2 (first used by B. Li in the study of isometric dilations of contractive representations of P on Hilbert space), and propagate positivity from J_1, J_2 to J. Use lists $\lambda : \{1, \dots, n\} \to P$ to accommodate repetitions in J_1, J_2.

A combinatorial tree for a list λ



For $\lambda(\{1,2,\dots,n\}) = \{p_1, p_2, \dots, p_n\}$, i first with $a \leq \lambda(i)$, set

$$\lambda_1(j) := \begin{cases} \lambda(j) & j \neq i \\ a & j = i; \end{cases} \quad \lambda_2(j) := \begin{cases} a^{-1}(a \vee \lambda(j)) & a \vee \lambda(j) < \infty \\ \infty & a \vee \lambda(j) = \infty. \end{cases}$$

A combinatorial tree for Noetherian monoids

A list $\lambda : \{1, 2, \ldots, n\} \to P \cup \{\infty\}$ is a *leaf* if either its image consists of atoms of P or ∞, or else if it intersects P^* (making the alternate sum equal to 0).
A *branch* is a finite or infinite word ω on the symbols $\{1, 2\}$ that starts at the root λ and ends at the first node such that the list after k-"steps" $\lambda_{\omega[1,k]}$ is a leaf, or does not end (if such a node does not exist).

Theorem (Gazdag-Laca-L (2022))

Let P be Noetherian right LCM. Suppose that the tree constructed above is finite for every list λ. Then reduction of positivity $(*)$ to positivity $(**)$ for subsets of atoms holds.

Proposition (GLL)

The tree of any list is finite for $P = A_M^+$ finite-type (since it is lattice ordered) or right-angled (since word length decreases along the tree).

KMS gaps for Artin monoids

Suppose $n \geq 3$ and let B_n be the braid group with generating set $S = \{s_1, s_2, \ldots, s_{n-1}\}$ and relations

$$\begin{aligned} s_i s_j s_i &= s_j s_i s_j && \text{when } |i-j| = 1 \\ s_i s_j &= s_j s_i && \text{when } |i-j| \geq 2. \end{aligned}$$

Let B_n^+ be the braid monoid and $N : B_n^+ \to [1, \infty)$ be given by

$$N(p) = \exp(\ell(p)), \ell(p) = |p|,$$

for $p \in B_n^+$. Let $\sigma = \sigma^N$. The clique polynomial

$$\sum_{K \in Cl(S)} (-1)^{|K|} t^{|\vee K|}$$

of $P = B_n^+$ is the reciprocal of the growth function $\sum_{n \geq 0} \#\{w \mid |w| = n\} t^n$.

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KMS gaps for Artin braid monoids

Proposition (GLL)

Let $t_1 = \sqrt{5}/2 - 1/2 \approx 0.618$ *be the smallest positive root of the clique polynomial* $1 - 2t + t^3$ *of* B_3^+, *and put* $b_1 = -\log t_1$. *Then the inverse temperature space of* $(C^*(B_3^+), \sigma)$ *is* $\{0\} \cup [b_1, \infty]$.

Proposition (GLL)

Let r_1 *be the smallest positive root of the polynomial* $1 - 2t - t^2 + t^3 + t^4 + t^5$ *of* B_4^+, *and* $c_1 = -\log r_1$. *Then the inverse temperature space of* $(C^*(B_4^+), \sigma)$ *is* $\{0\} \cup [c_1, \infty]$.

KMS gaps for Artin braid monoids

Thus:

1. $(C^*(B_3^+), \sigma)$ has inverse temperature space $\{0\} \cup [b_1, \infty]$, for e^{-b_1} the smallest positive root of $1 - 2t + t^3$.
2. $(C^*(B_4^+), \sigma)$ has inverse temperature space $\{0\} \cup [c_1, \infty]$, for e^{-c_1} the smallest positive root of $1 - 2t - t^2 + t^3 + t^4 + t^5$.

Sketch of proof: known from Bruce-Laca-Ramagge-Sims that if a KMS$_\beta$-state exists for some $\beta \in (0, \beta_0)$, then $e^{-\beta}$ has to be a root of the clique polynomial in the interval $(e^{-\beta_0}, 1)$. Then test the positivity criterion $(**)$ for finite-type A_M^+ to see if any such root leads to equilibrium states. The positivity fails at intermediate roots in the interval.

Conclusion: Searching for KMS$_\beta$ states leads to looking quite closely into the structure of the monoid. Other monoids could be worth considering.

OAMN - on behalf of Kristin Courtney

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Final

THANK YOU!